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Kill Vehicle Guidance and Control Sizing for Boost-Phase Intercept

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This paper addresses some of the guidance and control issues involved in enabling an air-launched interceptor carrying a highly maneuverable kinetic kill vehicle to perform an exoatmospheric intercept of a boosting threat missile capable of traveling many thousands of kilometers. The paper takes the reader through the first iteration of the multi-iteration design process in order to show how much divert and acceleration are required by the kinetic kill vehicle to hit the target. Simplified examples are presented to indicate how conventional guidance and filtering techniques can be used as a starting point in the iterative design process for this important problem in missile defense. More advanced guidance and filtering techniques can be used in subsequent iterations to more accurately size the kinetic kill vehicle and improve system performance and robustness.

Introduction

INTERCEPT of intercontinental ballistic missiles (ICBMs) during their boost phase has long been considered attractive, even to critics of missile defense, because boosting missiles are easy to see and decoys are difficult to employ as countermeasures. However, a few years ago, the American Physical Society (APS) released a detailed report [1] that studied the use of surface-based interceptors for intercepting ICBMs during their boost phase. The APS report found that the surface-based interceptors required for boost-phase intercept would have to be very heavy due to both the high required burnout velocities and lateral divert requirements of the interceptor. In addition, in the case of an Iranian ICBM launch against the United States, the APS report showed that the interceptors would have to be based in countries that might present a political challenge for the United States. Thus, in general, the APS report was very pessimistic about the success of a terrestrial-based boost-phase intercept system.

Air-Launched Interceptor Approach

An alternative approach to boost-phase intercept involves the use of airborne interceptors and was first considered in the open literature by Wilkening [2] and then expanded upon, with considerable practical detail, by Corbett and Zarchan [3]. At first glance, this alternative approach to boost-phase intercept might be considered inferior to surface-based interceptors, since airborne interceptors would have even lower burnout velocities than surface-based interceptors due to aircraft payload weight constraints. However, in this alternative approach, the stealthy aircraft would initially be manned, but in the far term, it would be unmanned, using a platform such as the Naval Unmanned Combat Air System Carrier [4]. Stealthiness would enable the aircraft to penetrate enemy territory to get much closer to ICBM launch sites than would be possible with surface-based interceptors located near the borders of an enemy nation. Having the defensive interceptor launch platform closer to an enemy launch site means that the required burnout velocity of an air-launched interceptor can be much less than that of a surface-launched interceptor.

The key elements in Corbett and Zarchan's boost-phase intercept system construct [3] are stealthy fighter aircraft with infrared search-and-track (IRST) systems to detect and track the target and airborne interceptors with highly maneuverable kinetic kill vehicles (KKVs) for exoatmospheric intercepts of the enemy missiles. In this system, construct pairs of stealthy aircraft travel in oval racetracks in opposite directions over enemy territory. Their combined IRST systems have 360 deg coverage and can search for, detect, and track enemy ICBMs and intermediate-range ballistic missiles (IRBMs) autonomously during their boost phase. When the IRST system of one aircraft detects a threat, it can cue another offboard IRST system to establish an additional angle-only track on the target so that the position, velocity, and acceleration of the target can be estimated. When sufficient track accuracy of the target states are obtained, a prediction is made of the target's position at the desired intercept time. This prediction will be imperfect, since it is impossible to know a boosting target's future intentions. The launch aircraft turns so that it can fire its interceptor directly at the predicted intercept point (PIP). The interceptor's thrust is not only used to increase the speed of the interceptor but, since the PIP is constantly changing, the interceptor thrust vector must also be steered in order for the interceptor to hit the latest and most refined estimate of the PIP. When the interceptor burns out, the PIP will still be in considerable error. Therefore, additional fuel and guidance is required so that a KKV, which separates from the interceptor after the interceptor burns out, will hit the target using its lateral divert engines for responding to guidance commands outside of the Earth's atmosphere.

Guidance and Control Issues

The purpose of this paper is to illustrate how some key guidance and control issues influence the amount of fuel and acceleration the KKV must have so that it can successfully engage both IRBMs and ICBMs. Sample tradeoffs will be conducted using conventional guidance and filtering methods to illustrate the first step of an iterative design process that must take place for all practical designs. Subsequent steps in the design process may consider more advanced guidance and filtering techniques that, in turn, might reduce the KKV divert requirements derived in this paper.

Lambert guidance is a conventional method of guidance when the control authority of the interceptor is in the axial direction, while augmented PN (APN) is appropriate when the control authority of the KKV is in the lateral direction [5]. Therefore, Lambert guidance can be used while the interceptor is thrusting, and APN can be used afterward by the KKV's lateral divert engines. The interceptor must be sized for both adequate burnout velocity and sufficient fuel and acceleration for the KKV's divert engines. When the KKV gets close enough to the target, its seeker can acquire the target plume. The

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seeker software must be capable of distinguishing the target hard body from the plume and enable the KKV to hit that target's warhead.

This paper starts out by first considering the effects of target maneuver, guidance law, and PIP error in a noise-free one-dimensional engagement environment. Formulas will be developed, showing how KKV divert requirements are related to PIP error, target maneuver, and KKV action time. Next, engagement experiments in two dimensions were conducted in a noise-free environment to see how simulation results compared with closed-form solutions. Finally, it is demonstrated that sensor noise and filtering effects also play an important role in establishing KKV lateral divert requirements.

One-Dimensional Model for Understanding Guidance

Figure 1 presents the classical interceptor homing loop for understanding guidance. Here, n_T represents the apparent target acceleration, as seen by the pursuing interceptor, of a boosting threat. That portion of the target's axial acceleration that is perpendicular to the KKV-target line of sight (LOS) will appear as a target maneuver to the KKV. In this example, we want to ensure that the interceptor has adequate acceleration capability (does not saturate near the end of the flight) so that it can hit the target. For exoatmospheric intercepts, the kill vehicle time constants are so small that they can be neglected in a preliminary analysis. In addition, we will also be concerned about the amount of KKV fuel or lateral divert required for a successful intercept.

Under worst-case geometrical conditions, all of the threat's axial acceleration will be seen by the interceptor as an apparent target maneuver. A typical acceleration profile for a generic one-stage 180 s burn IRBM is displayed as the solid curve in Fig. 2. Here, it can be seen that the threat acceleration increases with increasing time because, as the IRBM propellant burns, the weight of the IRBM decreases. In this example, the maximum acceleration of the IRBM is approximately 9 g at 180 s. For academic and analytical purposes, the sample IRBM acceleration can be initially approximated by a parabola with zero acceleration and finally approximately by $n_{T\text{MAX}}$ acceleration. The parabolic approximation is given by

$$n_{T_{\text{Parabola}}} = n_{T_{\text{Max}}} \left(\frac{t}{t_F} \right)^2$$

where t_F is the IRBM burnout time. It will be shown later that the real purpose of the parabolic approximation is to enable us to rapidly estimate the KKV divert and acceleration requirements on hypothetical threats when only a minimal amount of information is available.

Let us simulate the one-dimensional guidance system of Fig. 1, assuming the proportional navigation (PN) guidance law with an effective navigation ratio of three, for the academic case in which the target of Fig. 2 and the interceptor are both launched at time zero and intercept occurs at target burnout (180 s). Figure 3 shows that, in this case, the actual missile acceleration required by the PN guidance law at the end of the flight is 30 g or three times the maximum acceleration capability of the target. This interceptor acceleration requirement is the same as would be the case if there was a constant target maneuver [5]. It can also be seen from Fig. 3 that the parabolic target maneuver approximation yields nearly identical KKV requirements at the end of flight, indicating that the parabolic target maneuver approximation might be a good approximation for a boosting target being pursued by an interceptor employing PN guidance.

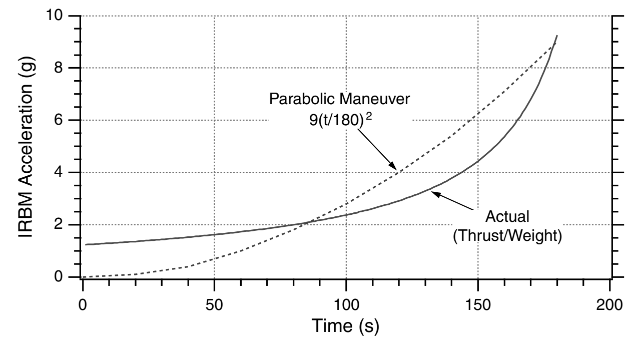

Fig. 2 For simplicity, the axial acceleration of an IRBM will be approximated by a parabola.

The one-dimensional simulation experiment was also repeated for the case in which the interceptor guidance law was changed to APN with an effective navigation ratio of three. The solid curve of Fig. 4 represents the acceleration required by the KKV using the APN guidance law against the boosting IRBM, whereas the dashed curve represents the required KKV acceleration due to the APN guidance law against a parabolic target maneuver approximation. From the solid curve of Fig. 4, it can be seen that the maximum acceleration required by the KKV against the actual boosting target is now only 3 g, or about one third of the maximum axial acceleration capability of the target, whereas the parabolic approximation indicates a 4.5 g maximum KKV acceleration, or half the maximum acceleration capability of the target. In addition, for both the actual and parabolic target acceleration models, the maximum acceleration no longer occurs at the end of flight. This means that, should the KKV acceleration saturate, there is a chance that it will come out of saturation and not cause miss distance. Thus, the maximum interceptor acceleration required by the APN guidance law is nearly an order of magnitude smaller than that required by the PN guidance law. This means that the performance improvement with APN is so great that we shall not even consider using PN guidance against a boosting target. Figure 4 also indicates that the parabolic approximation to the target maneuver is not as good in predicting interceptor performance as it was in the previous example where PN guidance was used by the KKV. However, the parabolic approximation does indicate that there is a dramatic performance improvement with APN guidance. It can be seen that using the parabolic approximation for the boosting

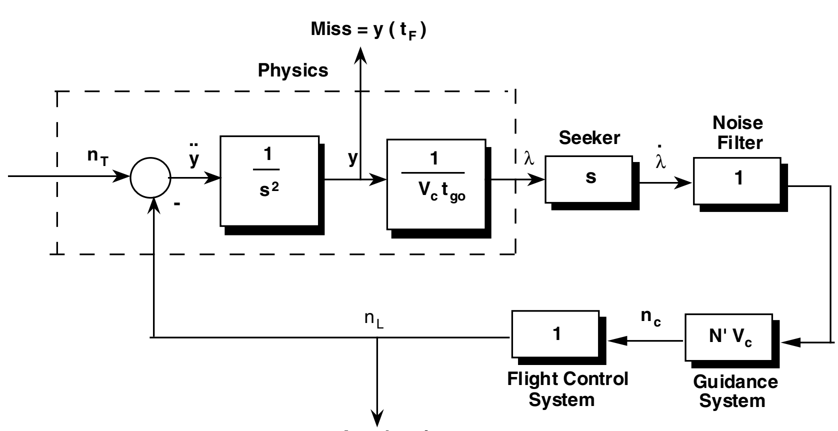

Fig. 1 One-dimensional guidance system model for initial analysis.

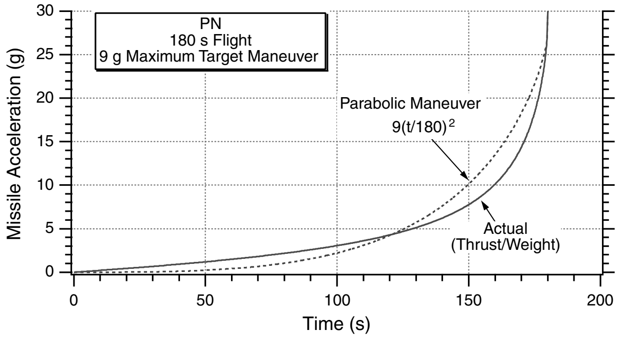

Fig. 3 For the PN guidance system, parabolic maneuver approximation yields accurate performance projections.

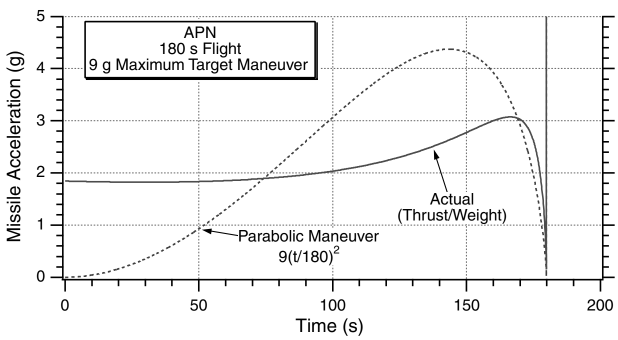

Fig. 4 For the APN guidance system, parabolic maneuver approximation to boosting target overestimates KKV acceleration requirements.

IRBM tends to overestimate the KKV acceleration requirements. Therefore, for a conservative starting point, the parabolic approximation to the boosting target might be appropriate for analysis because of its simplicity.

Developing Formulas for Divert due to Boosting Target and Predicted-Intercept-Point Errors

Generally speaking, an exoatmospheric interceptor has to be used against a long-range boosting target, because most of the target's flight is outside the atmosphere. An endoatmospheric interceptor may be required against short-range ballistic missiles because their apogee is very low. In this paper, only long-range IRBMs and ICBMs are considered. The KKV part of the interceptor has lateral divert engines for implementing guidance commands outside of the atmosphere. Since the divert engines burn propellant to implement the guidance law, guidance terminates when the propellant is expended, since the KKV can no longer maneuver. The amount of propellant required by the KKV is related to the lateral divert ΔV through the rocket equation. Thus, the amount of lateral divert required for an intercept is an important measure of interceptor performance. The lateral divert is simply the integral of the absolute value of the interceptor acceleration or

$$\Delta V = \int_0^{t_F} |n_c| \, \mathrm{d}t$$

Figure 1 was evaluated for a 10 g parabolic target maneuver and the APN guidance law for KKV flight times ranging from 10 to 50 s in steps of 10 s. The simulation results of Fig. 5 indicate that the amount of lateral divert required increases linearly with KKV flight time. We can also see from Fig. 5 that the simulation results can be curve fitted, and an empirical formula can be developed for the KKV lateral divert due to a parabolic target maneuver as

$$\Delta V_{\mathrm{MVR}} = 0.25 n_{T_{\mathrm{Max}}} t_F$$

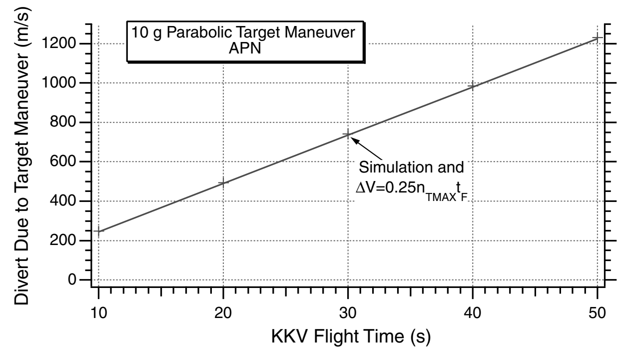

Fig. 5 A formula for KKV divert, due to parabolic target maneuver in the APN guidance system, can be developed.

where $n_{T\mathrm{Max}}$ is the maximum value of the parabolic maneuver in units of meters per second squared $(\mathrm{m/s^2})$ and ΔV_{MVR} is in units of meters per second (m/s). The quantity t_F represents the KKV action time or the amount of time the KKV is maneuvering.

Up to this point, it has been assumed that the only disturbance entering the KKV guidance system was an apparent target maneuver. Another important error source is the PIP error. This error source is due to the fact that the location of the boosting target at the desired intercept time is unknown. A prediction of the intercept point must be made, and this prediction will have errors. The errors will be the same, whether a PN or APN guidance law is used.

It is important to note that some people believe that the PIP errors will be small because a priori information concerning the threat will be available. The point of view taken in this paper is that, even if the thrust–weight profile of the threat was known perfectly, the future direction of the acceleration vector is unknown. In other words, we do not know where the target is going or when it will get there based on past information. Therefore, the conservative point of view taken in this paper is that the system must work when information concerning the threat is not available or denied and that the lateral divert of the KKV must be sized accordingly. It can be shown [5] that for the model of Fig. 1, the KKV lateral divert due to the PIP error for the APN guidance law with an effective navigation ratio of three is given by

$$\Delta V_{\mathrm{PIP}} = 1.5 \frac{\mathrm{PIP}}{t_F}$$

Here it can be observed that, for a given PIP error, more divert will be required for shorter KKV action times t_F. Thus, the total divert required, under worst-case conditions, is simply given by

$$\Delta V_{\mathrm{TOT_{APN}}} = \frac{1.5 \cdot \mathrm{PIP}}{t_F} + 0.25 \cdot n_{T\mathrm{MAX}} t_F$$

Interceptor–Intermediate-Range Ballistic Missile Engagements

The previously mentioned single-stage generic IRBM model was put in a two-dimensional nonlinear engagement simulation. A sample 2000 km lofted trajectory of the IRBM is presented in Fig. 6. We can see that the IRBM apogee in this case is approximately 500 km.

The boost-phase portion, or the first 180 s of the IRBM trajectory, is presented with 10 s time tics in Fig. 7. If it is assumed that there is cloud cover until 7 km altitude, then the IRBM can be seen by airborne IRST sensors at 80 s. Of course, on a clear day, the target can be seen much sooner.

Assuming that 10 s are required by the aircraft IRST sensors to establish a firm track on the IRBM, then the earliest an interceptor can be launched would be at 90 s. Figure 8 depicts an air-launched 4 km/s interceptor with a 20 s burn phase being launched at 90 s and 15 km altitude at the PIP. In this scenario, an intercept is to occur at 170 or 10 s before the IRBM burns out. The case in which the PIP is

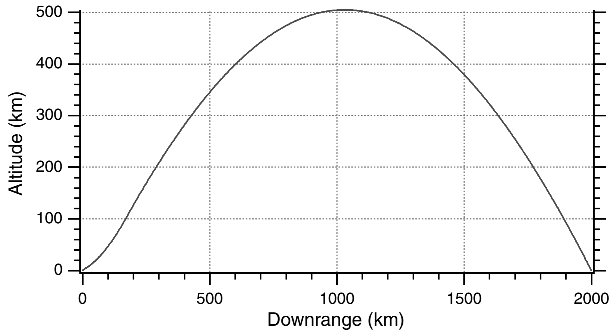

Fig. 6 Sample 2000 km IRBM trajectory.

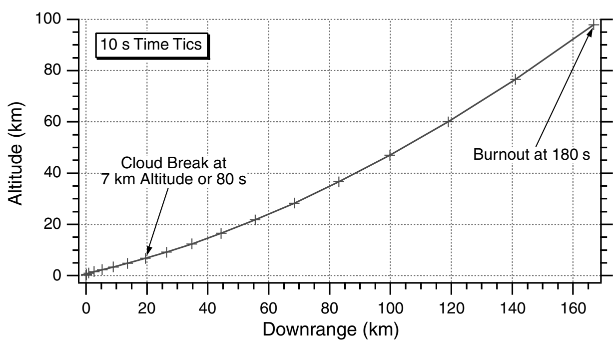

Fig. 7 Boost-phase portion of IRBM trajectory.

known perfectly in order to establish KKV divert requirements due to an apparent target maneuver is considered first. Next, a postburnout intercept, where there is no apparent target maneuver, is investigated in order to establish KKV divert requirements due to PIP error.

Let us examine the case in which there is no PIP error. In this exercise, the interceptor would not even require a KKV, since it can fly directly toward the perfect PIP. However, as part of this academic exercise, the KKV guidance system, using the APN guidance law, is turned on at 110 s (20 s after interceptor launch and when the interceptor has reached its burnout velocity of 4 km/s). Since the portion of the IRBM acceleration that is perpendicular to the LOS appears as a target maneuver to the pursuing interceptor, the KKV will maneuver in order to hit the apparently maneuvering target. The KKV maneuvering will occur even though the PIP is known perfectly. The interceptor–target engagement geometry for this example is displayed in Fig. 8.

The required KKV acceleration to hit the target along with the target acceleration perpendicular to the LOS is displayed in Fig. 9. Here it can be observed that, for this engagement geometry, the maximum target acceleration perpendicular to the LOS is approximately 3.6 g, and the maximum KKV acceleration required by the APN guidance law is approximately 0.8 g, or approximately four times less acceleration than the target. The amount of lateral divert

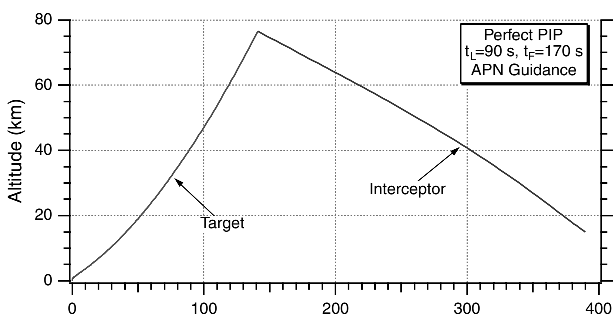

Fig. 8 Engagement with interceptor launched 10 s after cloud break and intercept occurring 10 s before the end of the IRBM boost phase.

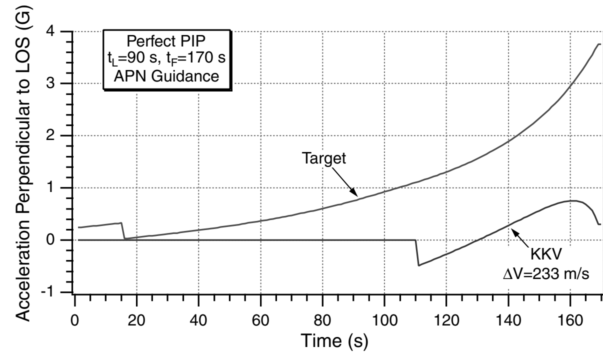

Fig. 9 For maximum range engagement, KKV requires much less acceleration than target.

required by the KKV, as indicated by the engagement simulation, is 233 m/s in this example.

If the divert formula based on the parabolic target maneuver is used, it can be seen that the theoretical KKV divert predicted is 540 m/s or

$$\Delta V_{\mathrm{MVR}} = 0.25 n_{T\mathrm{MAX}} t_F = 0.25 \cdot 36 \cdot (170 - 110) = 540 \text{ m/s}$$

As expected, the theoretical KKV divert prediction is observed to be very conservative, since it is about twice as high as the actual KKV divert required, as indicated by the engagement simulation. Generally speaking, it was found that the KKV divert due to the apparently maneuvering one-stage IRBM target, as indicated by the engagement simulation, for different initial separations between both the interceptor and target launch points was usually less than a few 100 meters per second and is thus considered negligible.

A much more important error source against this one-stage IRBM target is the PIP error. Figure 10 shows a case in which there is a 63 km PIP error at the end of the interceptor burn phase. In this academic exercise, the KKV guidance system is turned on after the target burns out (so that none of the KKV divert is due to the apparent target maneuver), and an intercept geometry is set up so that the desired intercept time is 230 s, but the achieved intercept time in the nonlinear engagement simulation turns out to be 223 s. To accommodate the postboost intercept time and to ensure that the intercept geometry was kinematically feasible, the interceptor launch point was moved further downrange. The engagement geometry for the PIP experiment is depicted in Fig. 10.

Figure 11 displays the KKV acceleration profile for the engagement with the 63 km PIP error and no apparent target maneuver. It can be seen that when the KKV guidance system turns on at 180 s, there is an immediate step in KKV acceleration. After the initial step, the acceleration linearly decreases to zero, which is in accordance with theory. The lateral divert required by the interceptor, as indicated by the engagement simulation, is 1.82 km/s.

A formula was previously provided for the KKV divert due to PIP error. For the case in which there was 63 km of PIP error and the time to take it out was 43 s ($223 - 180 = 43$), the theoretical divert

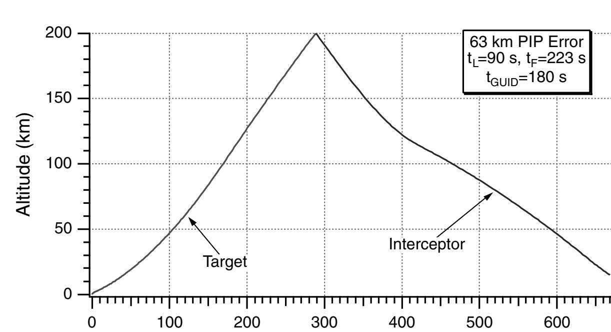

Fig. 10 Postboost phase engagement to test formula for PIP error.

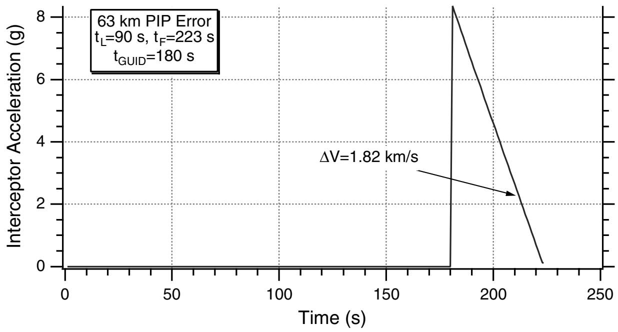

Fig. 11 Large initial KKV acceleration is required to take out PIP error in a short time.

formula indicates that 2.09 km/s of KKV lateral divert is required, or

$$\Delta V_{\mathrm{PIP}} = 1.5\frac{\mathrm{PIP}}{t_F} = 1.5\frac{63,000}{(223-180)} = 2.09 \text{ km/s}$$

which is close to the engagement simulation results of Fig. 11.

Simple Method for Calculating Predicted Intercept Point

For the one-stage IRBM, it has been observed that the dominant contributor to the KKV lateral divert can be the PIP error. However, we have not discussed how the PIP is calculated. One simple way of predicting where the target will be at intercept is to use a three-term Taylor series based on the current position, velocity, and acceleration of the target, or in two dimensions,

$$x_F = x + \dot{x}t_{\mathrm{go}} + 0.5\ddot{x}t_{\mathrm{go}}^2, \qquad y_F = y + \dot{y}t_{\mathrm{go}} + 0.5\ddot{y}t_{\mathrm{go}}^2$$

where t_{go} is given by

$$t_{\mathrm{go}} = t_F - t$$

In other words, the future location of the object at some time t_F is the current target position plus the current target velocity multiplied by the time to go plus one half the current target acceleration multiplied by the time to go squared. The Taylor series method of prediction has many faults, but its main virtue is that it does not require a priori information.

The scenario of Fig. 8 was rerun for several cases where the interceptor launch point was gradually moved closer to the target launch point. Recall that the interceptor launch time is 90 s, and the KKV starts guidance at 110 s. In each case, the earliest boost-phase intercept time, as dictated by the kinematics, was selected. The simulated and calculated divert results are displayed in Fig. 12. It is important to note that the simulated results include the effect of the apparent target maneuver, whereas the divert formula does not. We can see that the theoretical divert formula due to PIP error slightly underestimates the total required KKV lateral divert, as indicated by the simulation. This means that for the one-stage IRBM example,

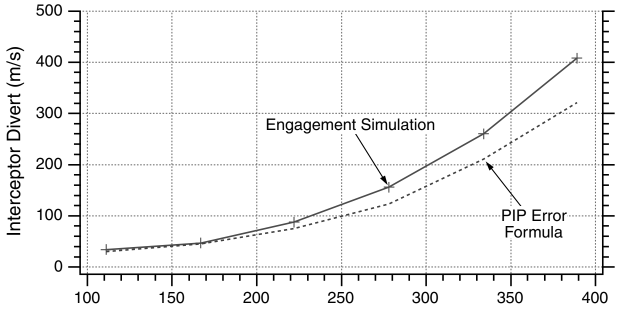

Fig. 12 Most of the KKV divert is due to PIP error for one-stage IRBM.

most of the KKV divert is due to the PIP error rather than the apparent target maneuver.

Interceptor–Intercontinental Ballistic Missile Engagements

Next a two-stage liquid ICBM threat with a 240 s burn time, as described in the APS report [1], was considered as the target for analysis. A sample lofted at a 10,000 km trajectory for the ICBM is depicted in Fig. 13, where it can be seen that the apogee of this trajectory is approximately 2000 km. The boost-phase portion, or first 240 s of the ICBM trajectory, is displayed in Fig. 14 with 20 s time tics. It can be observed that the ICBM breaks the clouds, assuming a 7 km altitude cloud cover, at approximately 60 s.

The total axial acceleration of the ICBM is depicted in Fig. 15. Here it can be seen that the first staging event occurs at 120 s, with a maximum acceleration of 6 g, and the second staging event occurs at 240 s, with a maximum acceleration of nearly 13 g. If the interceptor is launched early and the KKV guidance starts before 120 s, then the KKV guidance system will experience a large step in target acceleration. It is apparent that this complex apparent acceleration cannot be represented by a parabola.

An engagement was set up in which the initial interceptor launch point was about 900 km from the target launch site. The interceptor

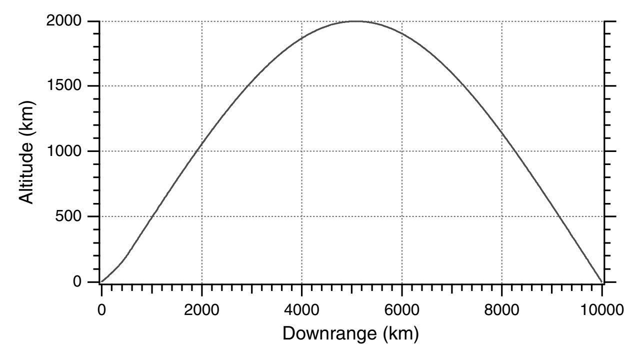

Fig. 13 Sample 10,000 km ICBM trajectory.

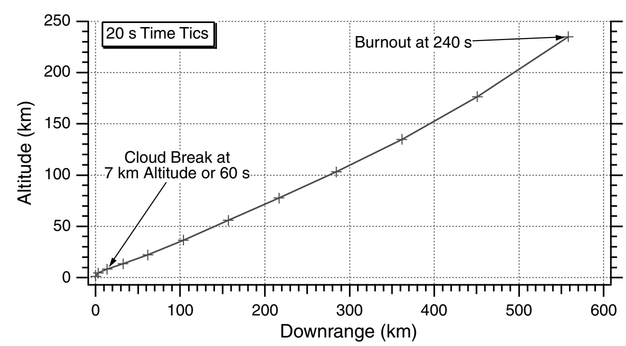

Fig. 14 Boost-phase portion of ICBM trajectory.

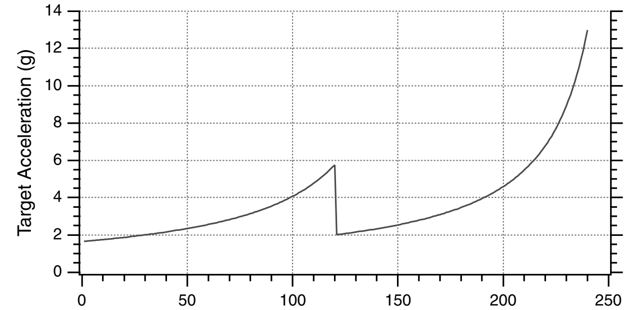

Fig. 15 ICBM has two staging events.

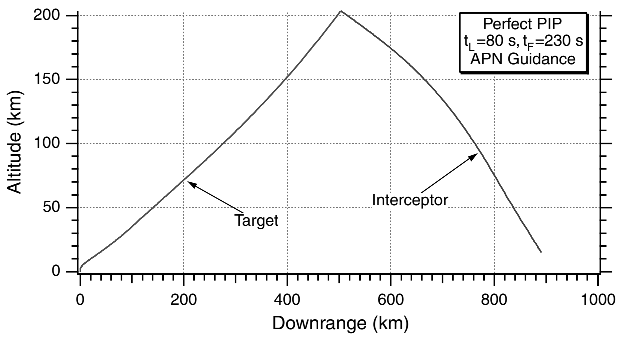

Fig. 16 ICBM engagement with interceptor launched 20 s after cloud break and intercept occurring 10 s before end of boost.

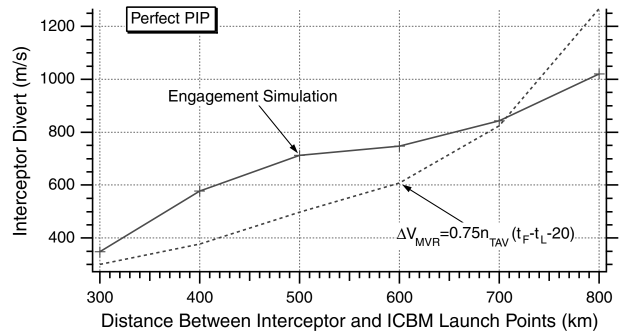

Fig. 18 Divert due to apparent constant target maneuver formula captures trends of actual KKV divert due to boosting two-stage ICBM.

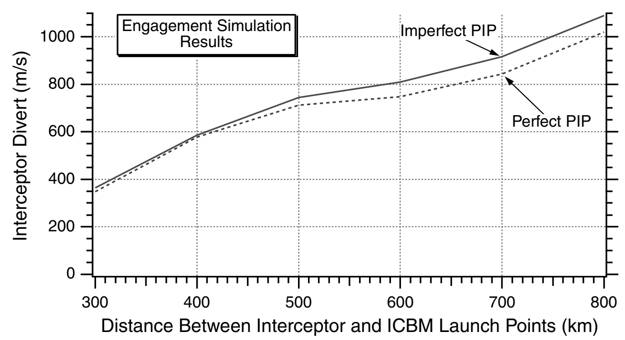

Fig. 19 Influence of PIP errors on KKV lateral divert for ICBM intercept is small.

was launched at 80 s, and the KKV guidance system using the APN guidance law was initiated at 100 s, or about 20 s before the first target staging event. The engagement geometry is depicted in Fig. 16.

Figure 17 displays the target axial acceleration that is perpendicular to the LOS (dashed curve) for the engagement of Fig. 16. As was mentioned previously, this acceleration projection appears as a target maneuver to the interceptor. The resultant KKV acceleration in response to the apparent target maneuver (solid curve) is also displayed in Fig. 17. It can be seen that, at first, the KKV acceleration closely follows the apparent target maneuver and then changes abruptly when the target goes through the staging event at 120 s. The magnitude of the KKV acceleration, after the staging event, then becomes a fraction of the actual target acceleration perpendicular to the LOS. The resultant lateral divert required by the KKV is shown in Fig. 17 to be 1020 m/s.

The average acceleration of the target maneuver while the KKV guidance system is activated can be computed from the target acceleration perpendicular to the LOS and is shown in Fig. 17 to be 1.3 g. If we pretend that the average target acceleration represents the complex apparent target maneuver shown in Fig. 17, then we can calculate the theoretical KKV divert (assuming APN guidance with an effective navigation ratio of three) to be [5]

$$\Delta V_{\mathrm{APN}} = 0.75 n_{T_{\mathrm{AV}}} t_F = 0.75 \cdot 13 \cdot (230 - 100) = 1268 \text{ m/s}$$

which is within 25% of the value 1020 m/s, shown in Fig. 17.

An important purpose of the preceding divert formula is to qualitatively explain the simulation results and to suggest that the required KKV divert increases with increasing homing time. The formula also suggests that launching the interceptor earlier may increase the KKV divert requirements for a given intercept time. Thus, an important purpose of the divert formula is to suggest future simulation experiments that must be conducted as part of the iterative design process.

Cases were run in which the interceptor downrange location was varied when the PIP was known perfectly. Figure 18 displays the comparison between the KKV divert from the engagement simulation results and the formula. It can be seen that the KKV divert

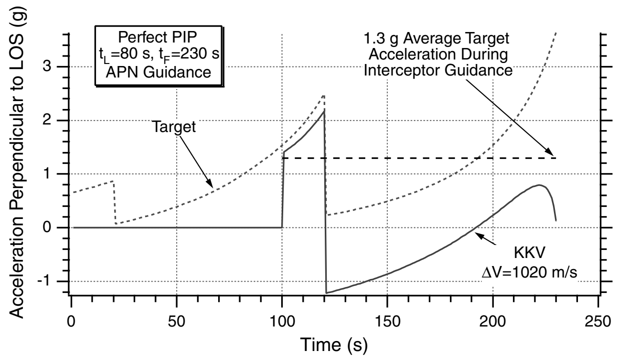

Fig. 17 For maximum-range ICBM engagement, apparent target acceleration is not a parabola.

trend is accurately captured with the simple divert formula appropriate for a constant average target maneuver.

Figure 19 shows another set of cases where the distance from the interceptor launch point to the target launch point is varied in the same way as was done in Fig. 18. However, this time, the PIP is calculated from a three-term Taylor series rather than being perfect. It can be seen from the engagement simulation results of Fig. 19 that for the two-stage ICBM case, the KKV divert requirements only increase slightly when PIP errors are considered. Thus, it can be concluded that for the ICBM case, the apparent target maneuver is the major contributor to the KKV divert requirements.

Noise and Filtering

Ideally, one would like to have an aircraft sensor that could measure both range and angle to the target so that we can estimate the target states required for calculating the PIP and for implementing Lambert guidance for the boosting interceptor and APN guidance for the KKV. Unfortunately, an airborne radar that can see the target at the long distances required might be too heavy. Similarly, an airborne laser detection and ranging may also not work at required distances. On the other hand, an IRST sensor can see the boosting target at great distances but can only measure angle. For angle-only tracking of an unpredictable target, triangulation or stereo tracking is generally required. To triangulate on the target, two aircraft are required (each having an IRST sensor), separated by a large distance known as a baseline. From the angle-only measurements of the two, sensors filters can be designed to estimate the position, velocity, and acceleration of the target. The triangulation of the angle measurements is also known as stereo tracking. One logical choice for filtering would be to design an extended Kalman filter (EKF) for this stereo tracking application.

Another choice would be to design an even simpler filter for the first step of the iterative design process, so that initial estimates of the increase in KKV divert requirements can be rapidly obtained. Such a choice might be the use of linear decoupled polynomial Kalman filters using pseudomeasurements. Although decoupled linear polynomial Kalman filters are not optimal in this stereo tracking application, they can easily and rapidly be designed by pretending the sensors are measuring distances to the target rather than angles from the sensor to the target. Later on, during subsequent stages of the iterative design and sizing process, more optimal filters such as the EKF can be considered to see if they can reduce the resultant KKV divert requirements.

The basis for the pseudomeasurements for the decoupled linear polynomial Kalman filters in two dimensions can be derived from Fig. 20. Here we see two sensors measuring angles θ_1 and θ_2. It is assumed that the location of the sensors (x_{s1}, y_{s1} and x_{s2}, y_{s2}) are known, but the location of the target (x_T, y_T) is unknown.

From Fig. 20, one can express the two sensor measurements of the angles θ_1 and θ_2 as

$$\theta_1 = \tan^{-1}\left(\frac{y_{s1} - y_T}{x_{s1} - x_T}\right)$$

$$\theta_2 = \tan^{-1}\left(\frac{y_{s2} - y_T}{x_{s2} - x_T}\right)$$

Here, we have two nonlinear equations with two unknowns. After some algebraic manipulation, one can solve for the coordinates of the target in terms of the angle measurements and the sensor locations as

$$x_T^* = \frac{x_{s2}\tan(\theta_2) - x_{s1}\tan(\theta_1) + y_{s1} - y_{s2}}{\tan(\theta_2) - \tan(\theta_1)}$$

$$y_T^* = y_{s1} - x_{s1}\tan(\theta_1) + \frac{x_{s2}\tan(\theta_2)\tan(\theta_1) - x_{s1}\tan^2(\theta_1) + (y_{s1} - y_{s2})\tan(\theta_1)}{\tan(\theta_2) - \tan(\theta_1)}$$

Thus, the two pseudomeasurements x_T^* and y_T^* will serve as inputs to the two decoupled three-state linear polynomial Kalman filters. One also has to develop formulas for the variance of the pseudomeasurement noise to be used by the Riccati equations in the Kalman filter. The variance of the pseudomeasurement noise on x_T can be found by using the chain rule from calculus. According to the chain rule,

$$\Delta x_T = \frac{\partial x_T}{\partial \theta_1}\Delta\theta_1 + \frac{\partial x_T}{\partial \theta_2}\Delta\theta_2$$

By squaring and taking expectations of both sides of the preceding equation, one can express the variance of the pseudomeasurement noise in terms of the variances of each of the IRST sensors as

$$\sigma_{x_T}^2 = \left(\frac{\partial x_T}{\partial \theta_1}\right)^2 \sigma_{\theta_1}^2 + \left(\frac{\partial x_T}{\partial \theta_2}\right)^2 \sigma_{\theta_2}^2$$

where the partial derivatives are evaluated as

$$\frac{\partial x_T}{\partial \theta_1} = \frac{(x_{s2} - x_{s1})\tan(\theta_2) + y_{s1} - y_{s2}}{\{[\cos(\theta_1)\tan(\theta_2) - \tan(\theta_1)]\}^2}$$

$$\frac{\partial x_T}{\partial \theta_2} = \frac{(x_{s1} - x_{s2})\tan(\theta_1) + y_{s2} - y_{s1}}{\{[\cos(\theta_2)\tan(\theta_2) - \tan(\theta_1)]\}^2}$$

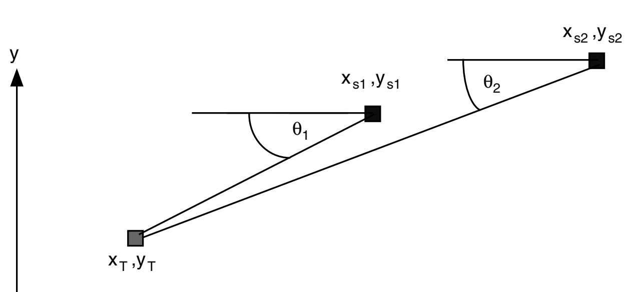

Fig. 20 Two angle-only sensors tracking a target.

The variance of the pseudomeasurement noise on y_T can be found in a similar way.

A Monte Carlo simulation was set up, where it was assumed that angle measurements were taken 10 times per second. The interceptor and KKV were command guided (first with Lambert guidance and then APN) until 10 s before intercept, based on the Kalman filter estimates. The two linear polynomial Kalman filter initial state estimates were set to zero (i.e., cold starting the filter), and within 10 s, accurate state estimates were obtained. During the last 10 s, it was assumed that homing guidance took place and that near-perfect estimates of the target states were available at a 100 Hz rate, so that APN could be implemented with range estimates being uplinked from the aircraft to the KKV.

A case was run in which the shooter aircraft (with sensor 1) was 800 km downrange of the target launch point and sensor 2 was 900 km downrange of the target launch point. Both sensors were 15 km in altitude. In addition, it was assumed that the angular accuracy of both sensors was 50 μrad and that the filter sampling time was 0.1 s. Figures 21 and 22 show how the linear polynomial Kalman filter is able to estimate the target acceleration for different values of filter process noise (Φ_s). Figure 22 shows that the filter with less process noise yields a much smoother estimate of target acceleration. However, by comparing the acceleration estimates to that of Fig. 21, one can see that the price paid for the smoother estimate is that the estimated target acceleration lags the actual target acceleration.

Frequently, in investigating Kalman filter performance, it is popular to look at the error in the estimates. Figure 23 shows how the single run error in the acceleration estimate for the filter with larger process noise ($\Phi_s = 576$) compares with the theoretical predictions provided by the Riccati equations. It can be seen that the single run results fall within the theoretical bounds, which indicates that the filter is consistent. On the other hand, Fig. 24 shows that when the process noise is reduced ($\Phi_s = 9$), the filter is no longer consistent. However, one can also see that the errors in the acceleration estimate of the filter with less process noise is smaller than when the process noise is increased. We shall soon see from a performance point of

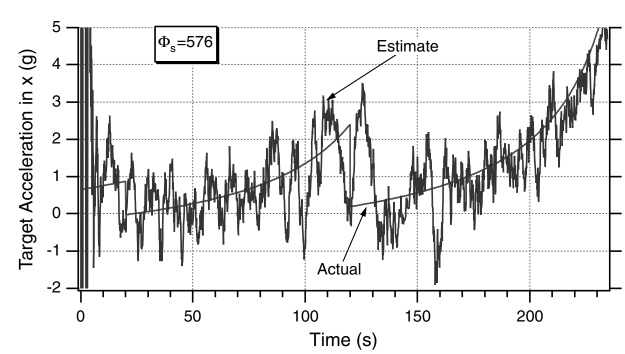

Fig. 21 Large Kalman filter process noise yields noisy estimates of target acceleration.

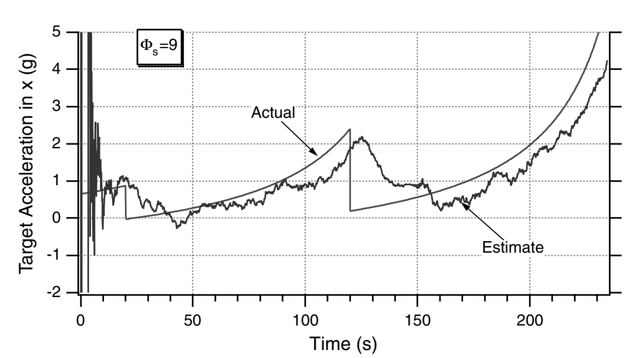

Fig. 22 Reducing Kalman filter process provides smoother estimates of target acceleration.

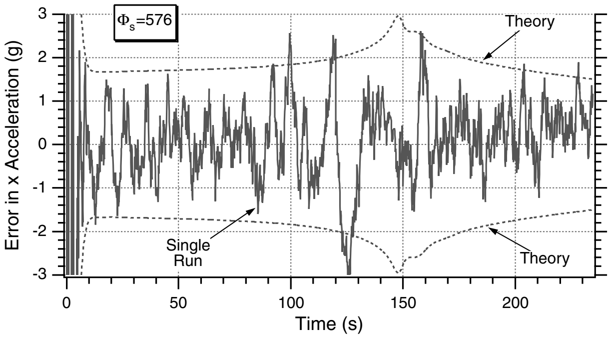

Fig. 23 Filter is consistent when process noise is high but estimation errors are large.

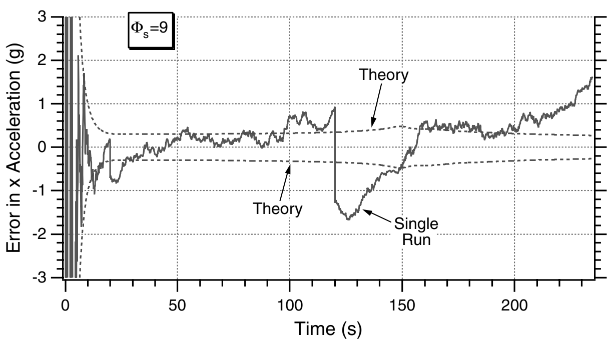

Fig. 24 Filter is not consistent when process noise is low but estimation errors are much lower.

view which filter is better for this particular boost-phase intercept problem.

Interceptor Engagements with Noise and Filtering

The engagement results for both the ICBM and IRBM targets were rerun, except this time with the sensor errors and Kalman filter state estimation. The goal was to see how much the inclusion of noise and filtering increased the KKV lateral divert requirements. The 90% point (i.e., 90% of the required divert was less than the amount shown in the following figures) in a Monte Carlo set was used as the figure of merit. Figure 25 shows that the KKV divert requirements still increase, with the initial distance from the interceptor launch point to the target launch site increasing. One can also see that sensor noise may substantially increase the KKV lateral divert requirements over the case where the state estimates are perfect. Figure 25 also shows that the selection of the amount of process noise that is used in the filter is important. Lower amounts of process noise may reduce the interceptor divert requirements, even though the filter is not consistent. However, even with a small value of process noise, the divert requirements can still increase more than 500 m/s over the no noise and filtering case. Thus, the inclusion of sensor noise and filtering is a very important part of the design process in setting KKV divert requirements. Subsequent iterations in the design and sizing process must include alternative filtering approaches, such as the EKF, to see if KKV divert requirements can be reduced.

The amount of sensor measurement noise is also very important. Figure 26 shows that if the sensor noise increases from its nominal value of 50 to 200 μrad, the lateral divert requirements can increase by more than 1000 m/s at the longer ranges. Thus, one can see that the guidance and control engineer must interact with the sensor designer to establish reasonable sensor requirements to achieve a balanced system design.

The IRBM engagement results were also repeated for the case in which noise and filtering were considered. One can see from Fig. 27 that including these realistic effects also increases the KKV lateral divert requirements. However, the KKV lateral divert requirements are much greater against the ICBM target than against an IRBM target. The reduced KKV divert requirements in this example are due to the fact that the one-stage IRBM burns for a shorter period of time; therefore, prediction errors are smaller than in the ICBM case. In addition, since there are no staging events, the apparent target maneuver with an IRBM is less severe than in the two-stage ICBM case. Thus, if we had to deal with both the IRBM and ICBM threats, the KKV lateral divert requirements would be set by the more difficult ICBM threat in this design process example. In practice, a wide variety of threats and possible trajectories would have to be considered in deriving KKV lateral divert requirements.

It has been mentioned that there are many steps in the iterative design process. In future steps, other sources of error must be considered to highlight potential problems and to suggest design work that must be performed. For example, in the analysis conducted so far, we have not considered measurement angle bias errors. Let us repeat the results of Fig. 25, where there is 50 μrad of measurement noise and no bias errors, to now include a 100 μrad angle bias error on the first IRST sensor. Figure 28 shows that the bias error increases the KKV lateral divert requirements. Methods for improving the KKV divert performance have to be explored. Thus, alternative filter structures must also be examined in subsequent iterations of the

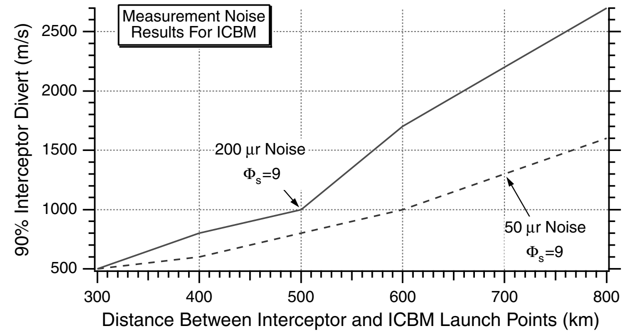

Fig. 26 KKV divert requirements increase with increasing measurement noise for the ICBM engagement.

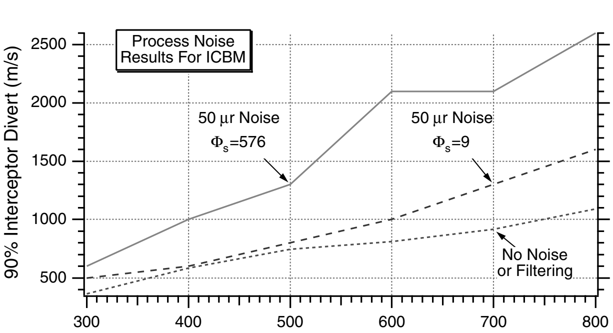

Fig. 25 Choice of process noise in Kalman filter can be very important in setting KKV divert requirements.

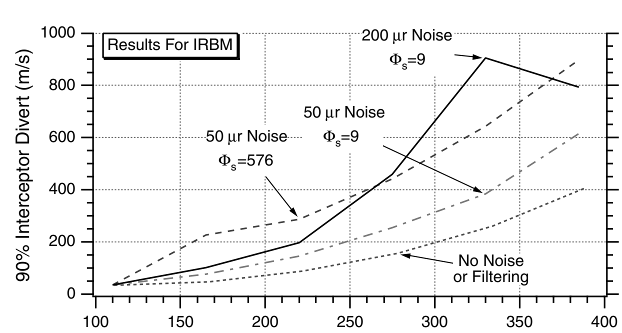

Fig. 27 KKV divert requirements also increase with increasing measurement noise for the IRBM engagement.

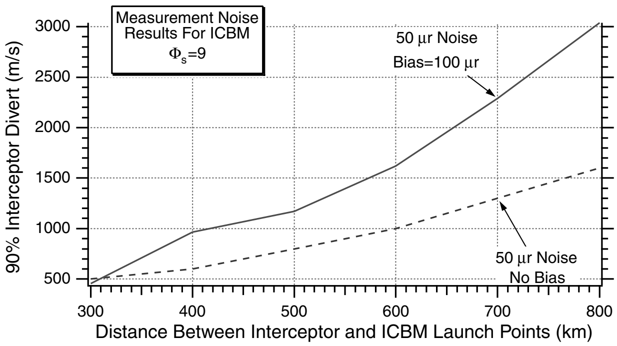

Fig. 28 Angle bias errors can increase KKV divert requirements or reduce maximum effective range of interceptor.

design process to see if the influence of biases can be alleviated. In future iterations of the design process, traditional approaches, such as redesigning the Kalman filter, increasing the homing time to avoid biases, placing more stringent design requirements on the IRST sensor, or including a star tracker on the aircraft, should be included for study. If no acceptable solutions to the bias error can be found, we can see that the effective range of the interceptor will be diminished. It is important to point out that problems are a normal part of the design process and that the search for solutions to these problems often leads to engineering innovation if the right people are involved.

Conclusions

This paper illustrates, through simplified examples, how airborne interceptor guidance and filtering technology are important in determining KKV lateral divert requirements against both boosting IRBM and ICBM threats. It was shown that with 50 μrad of aircraft IRST sensor noise and 2 km/s of KKV, divert boost-phase intercepts of both IRBM and ICBM targets could be achieved if the interceptor launching aircraft could be within 300 to 800 km of the target launch site. Traditional methods of guidance and filtering were employed to achieve these results. These methods were used to illustrate how sensor noise, prediction error, and apparent target maneuver work together in setting the KKV's lateral divert requirements. Simple formulas were developed that can be used to understand and explain engagement simulation results. Alternative guidance and filtering techniques have to be explored in subsequent iterations of the design process to see if the KKV divert and acceleration requirements can be reduced.

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